Subject Code: KAS103

PAPER ID-411938

Roll No:

BTECH
(SEM I) THEORY EXAMINATION 2021-22
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Notes:

- *Attempt all Sections and Assume any missing data.*
- *Appropriate marks are allotted to each question, answer accordingly.*

SECTION-A	Attempt **All** of the following Questions in brief	Marks**(10X2=20)**	**CO**
Q1(a)	Find the eigen value of A^3 where A=$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.		1
Q1(b)	Show that the system of vectors $X_1 = (1, -1, 1), X_2 = (2, 1, 1), and$ $X_3 = (3, 0, 2)$are linearly dependent or linearly independent.		1
Q1(c)	If $y = A \sin nx + B \cos nx$, prove that $y_2 + n^2 y = 0$.		2
Q1(d)	Find the asymptotes parallel to y-axis of the curve $\frac{a^2}{x} + \frac{b^2}{y} = 1$.		2
Q1(e)	If $x = r\cos\theta , y = r\sin\theta, find$ $\frac{\partial(r,\theta)}{\partial(x,y)}$.		3
Q1(f)	An error of 2% is made in measuring length and breadth then find the percentage error in the area of the rectangle.		3
Q1(g)	Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dy\, dx$.		4
Q1(h)	Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.		4
Q1(i)	Find p such that$\vec{A} = (px + 4y^2 z)i + (x^3 \sin z - 3y)j - (e^x + 4\cos x^2 y)k$ is solenoidal.		5
Q1(j)	State Green's theorem for a plane region.		5

SECTION-B	Attempt **ANY THREE** of the following Questions	Marks**(3X10=30)**	**CO**
Q2(a)	Find the eigen values and corresponding eigen vectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.		1
Q2(b)	Verify Rolle's theorem for the function $f(x) = \sqrt{4 - x^2}$ in $[-2, 2]$.		2
Q2(c)	Find the first six terms of the expansions of the function $e^x \log(1 + y)$ in a Taylor series in the neighborhood of the point $(0, 0)$.		3
Q2(d)	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.		4
Q2(e)	If a vector field is given by $\vec{F} = (x^2 - y^2 + x)i - (2xy + y)j$ Is this field irrotational? If so, find its scalar potential.		5

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	**CO**
Q3(a)	Find for what values of λ and μ the system of linear inequation: $x + y + z = 6$, $x + 2y + 5z = 10$,$2x + 3y + \lambda z = \mu$ has(i) a unique solution, (ii) no solution, (iii) infinite solution. Also find the solution for $\lambda = 2$ and $\mu = 8$.		1
Q3(b)	Find the rank of matrix reducing it to normal form $A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$		1

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SECTION-C	Attempt **ANY ONE** following Question — Marks **(1X10=10)**	**CO**
Q4(a)	If $y = (sin^{-1} x)^2$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ and calculate $y_n(0)$.	2
Q4(b)	Verify mean value theorem for the function $f(x) = x(x - 1)(x - 2)\ in\ \left[0, \frac{1}{2}\right]$.	2

SECTION-C	Attempt **ANY ONE** following Question — Marks **(1X10=10)**	**CO**
Q5(a)	A rectangular box which is open at the top having capacity 32c.c.Find the dimension of the box such that the least material is required for its constructions.	3
Q5(b)	If u, v and w are the roots of $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$, cubic in λ, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	3

SECTION-C	Attempt **ANY ONE** following Question — Marks **(1X10=10)**	**CO**
Q6(a)	Find by double integration the area enclosed by the pair of curves $y = 2 - x \quad and \quad y^2 = 2(2 - x)$.	4
Q6(b)	Find C.G. of the area in the positive quadrant of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.	4

SECTION-C	Attempt **ANY ONE** following Question — Marks **(1X10=10)**	**CO**
Q7(a)	Find the directional derivative of $f(x, y, z) = xyz$ at the point $P(1, -1, 2)$ in the direction of the vector $(2i - 2j + 2k)$.	5
Q7(b)	Verify Stoke's Theorem for $\vec{F} = (y - z + 2)i + (yz + 4)j - (xz)k$ over the surface of cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the XOY plane.	5