

B.Tech.
(SEM-I) THEORY EXAMINATION 2018-19
MATHEMATICS-I

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions.

Q no.	Question	Marks	CO
a.	Find the rank of the matrix $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$.	2	1
b.	Find the stationary point of $f(x, y) = x^3 + y^3 + 3axy, a > 0$	2	3
c.	If $x = r \cos \theta, y = r \sin \theta, z = z$ then find $\frac{\partial(r, \theta, z)}{\partial(x, y, z)}$	2	3
d.	Define del ∇ operator and gradient.	2	5
e.	If $\phi = 3x^2y - y^3z^2$, find grad ϕ at point (2, 0, -2).	2	4
f.	Evaluate $\int_0^1 \int_0^x e^x dx dy$	2	4
g.	If the eigen values of matrix A are 1, 1, 1, then find the eigen values of $A^2 + 2A + 3I$.	2	1
h.	Define Rolle's Theorem	2	2
i.	If $u = x^3y^2 \sin^{-1}(y/x)$, then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.	2	3
j.	In $RI = E$ and possible error in E and I are 20% and 10 % respectively, then find the error in R.	2	3
k.	State the Taylor's Theorem for two variables.	2	3

SECTION B

2. Attempt any three of the following:

Q no.	Question	Marks	CO
a.	Using Cayley- Hamilton theorem find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.	10	1

Also express the polynomial $B = A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$ as a quadratic polynomial in A and hence find B.

- b. If $y = \sin(m \sin^{-1}x)$, prove that : $(1 - x^2) y_{n+2} - (2n + 1)x y_{n+1} - (n^2 - m^2)y_n = 0$ and find y_n at $x = 0$. 2
- c. If u, v, w are the roots of the equation $(x - a)^3 + (x - b)^3 + (x - c)^3 = 0$, then find $\frac{\partial(u, v, w)}{\partial(a, b, c)}$. 3
- d. Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2 + y^2)} dx dy$ by changing to polar coordinates. Hence show that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. 4
- e. Verify the divergence theorem for $\vec{F} = (x^3 - yz)\hat{i} + (y^3 - zx)\hat{j} + (z^3 - xy)\hat{k}$, taken over the cube bounded by planes $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$. 5

SECTION C

3. Attempt any one part of the following:

- | Q no. | Question | Marks | CO |
|-------|--|-------|----|
| a. | Find inverse employing elementary transformation $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ | 10 | 1 |
| b. | Reduce the matrix A to its normal form when $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ | 10 | 1 |
- Hence find the rank of A.

4. Attempt any one part of the following:

- | Q no. | Question | Marks | CO |
|-------|--|-------|----|
| a. | If $\sin^{-1} y = 2 \log(x + 1)$ show that $(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$ | 10 | 2 |
| b. | Verify Lagrange's Mean value Theorem for the function $f(x) = x^3$ in $[-2, 2]$ | 10 | 2 |

5. Attempt any one part of the following:

- | Q no. | Question | Marks | CO |
|-------|--|-------|----|
| a. | Find the maximum or minimum distance of the point $(1, 2, -1)$ from the sphere $x^2 + y^2 + z^2 = 24$. | 10 | 3 |
| b. | If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$ | 10 | 3 |

6. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Change the order of integration and then evaluate: $\int_0^{23-x} \int_{\frac{x^2}{4}}^{xy} xy \, dy \, dx$.	10	4
b.	Calculate the volume of the solid bounded by the surface $x=0$, $y=0$, $x+y+z=1$ & $z=0$.	10	4

7. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both Solenoidal and Irrotational.	10	5
b.	Find the directional derivative of $\Phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point $P(1, 1, 1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.	10	5